Fuzzy logic

Introduction

Fuzzy Inference

Contents

- Fuzzy Inference
  - Fuzzification of the input variables
  - Rule evaluation
  - Aggregation of the rule outputs
  - Defuzzification
- Mamdani
- Sugeno

Fuzzy Inference

- The most commonly used fuzzy inference technique is the so-called **Mamdani** method.

- In 1975, Professor Ebrahim Mamdani of London University built one of the first fuzzy systems to control a steam engine and boiler combination. He applied a set of fuzzy rules supplied by experienced human operators.

Mamdani Fuzzy Inference

- The Mamdani-style fuzzy inference process is performed in four steps:

1. Fuzzification of the input variables
2. Rule evaluation (inference)
3. Aggregation of the rule outputs (composition)
4. Defuzzification.

Mamdani Fuzzy Inference

We examine a simple two-input one-output problem that includes three rules:

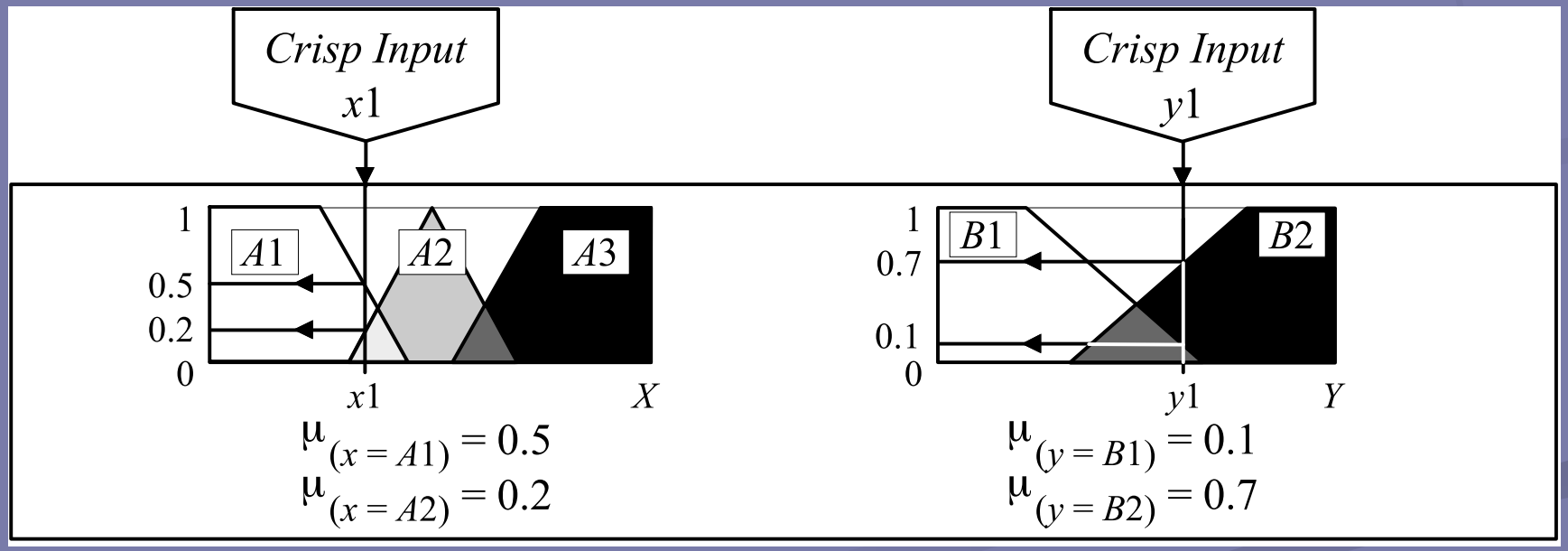
Rule: 1	IF x is A3	OR	y is B1	THEN	z is C1
Rule: 2	IF x is A2	AND	y is B2	THEN	z is C2
Rule: 3	IF x is A1			THEN	z is C3

Real-life example for these kinds of rules:

Rule: 1	IF project_funding is adequate	OR	project_staffing is small	THEN	risk is low
Rule: 2	IF project_funding is marginal	AND	project_staffing is large	THEN	risk is normal
Rule: 3	IF project_funding is inadequate			THEN	risk is high

Step 1: Fuzzification

- The first step is to take the crisp inputs, x1 and y1 (project funding and project staffing), and determine the degree to which these inputs belong to each of the appropriate fuzzy sets.



Step 2: Rule Evaluation

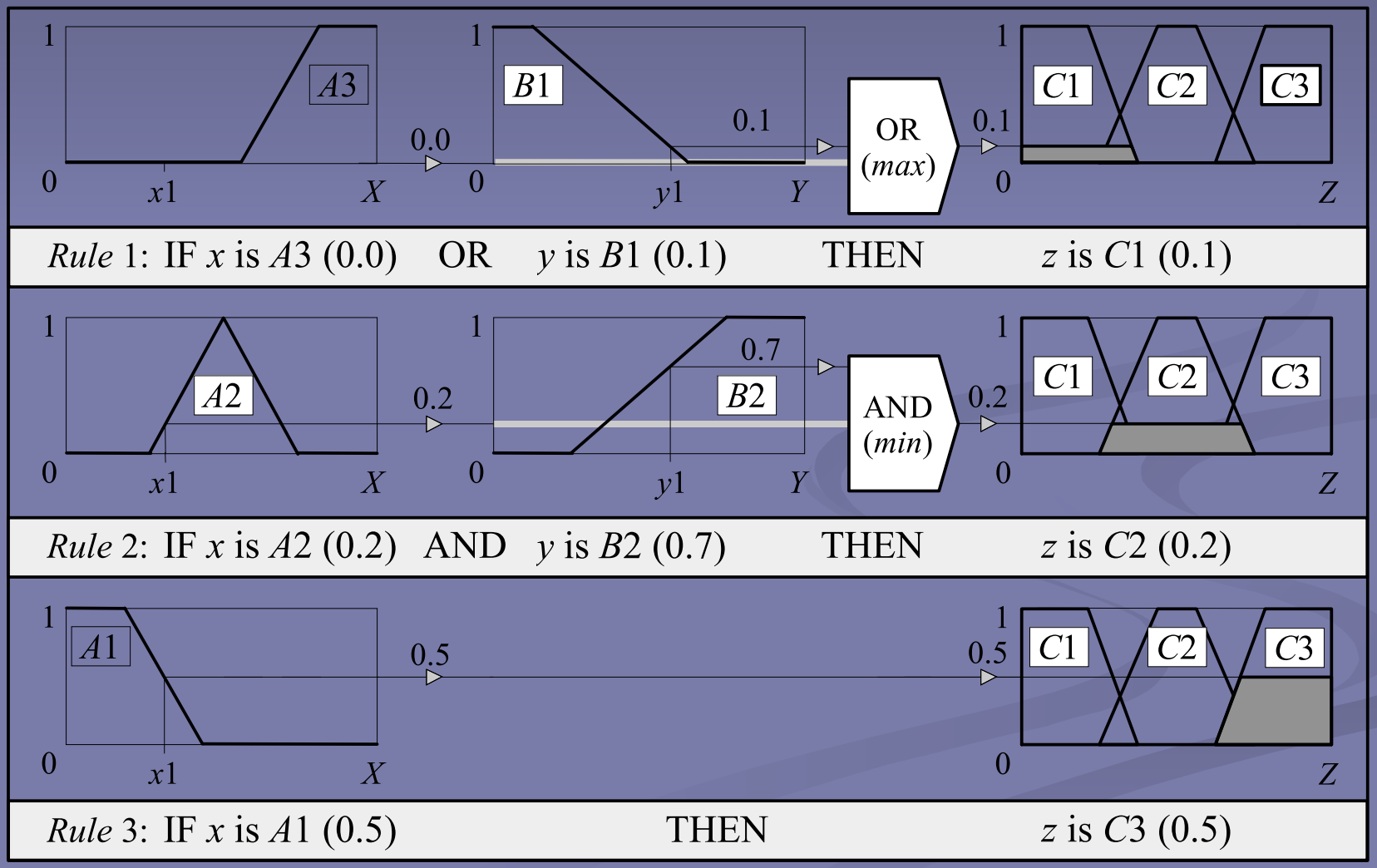
- The second step is to take the fuzzified inputs, $\mu_{(x=A1)} = 0.5$, $\mu_{(x=A2)} = 0.2$, $\mu_{(y=B1)} = 0.1$ and $\mu_{(y=B2)} = 0.7$, and apply them to the antecedents of the fuzzy rules.
- If a given fuzzy rule has multiple antecedents, the fuzzy operator (AND or OR) is used to obtain a single number that represents the result of the antecedent evaluation.

RECALL: To evaluate the disjunction of the rule antecedents, we use the **OR** fuzzy operation. Typically, fuzzy expert systems make use of the classical fuzzy operation union:

$$\mu_{A \cup B}(x) = \max [\mu_A(x), \mu_B(x)]$$

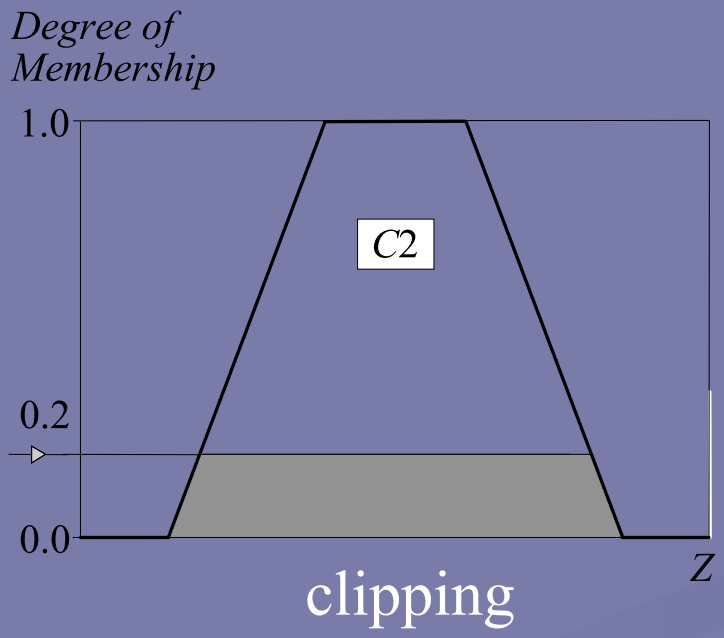
Similarly, in order to evaluate the conjunction of the rule antecedents, we apply the **AND** fuzzy operation intersection:

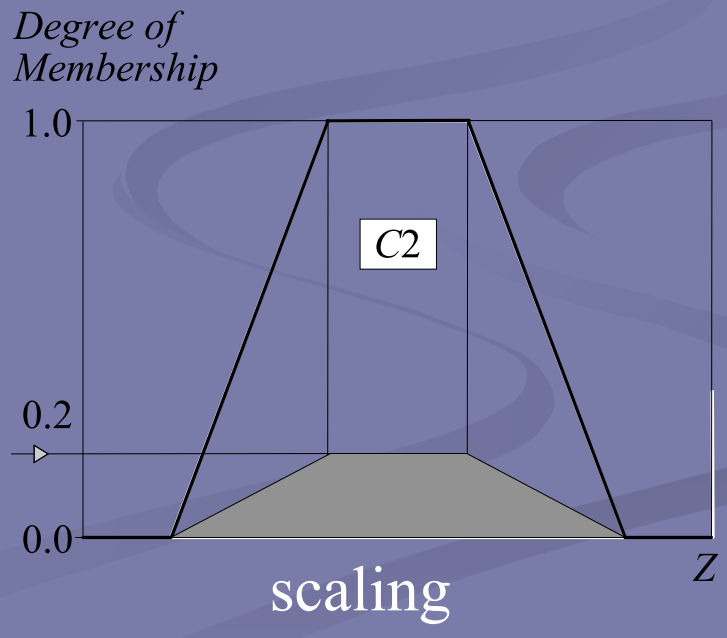
Step 2: Rule Evaluation



Step 2: Rule Evaluation

- Now the result of the antecedent evaluation can be applied to the membership function of the consequent.
- There are two main methods for doing so:
  - Clipping
  - Scaling



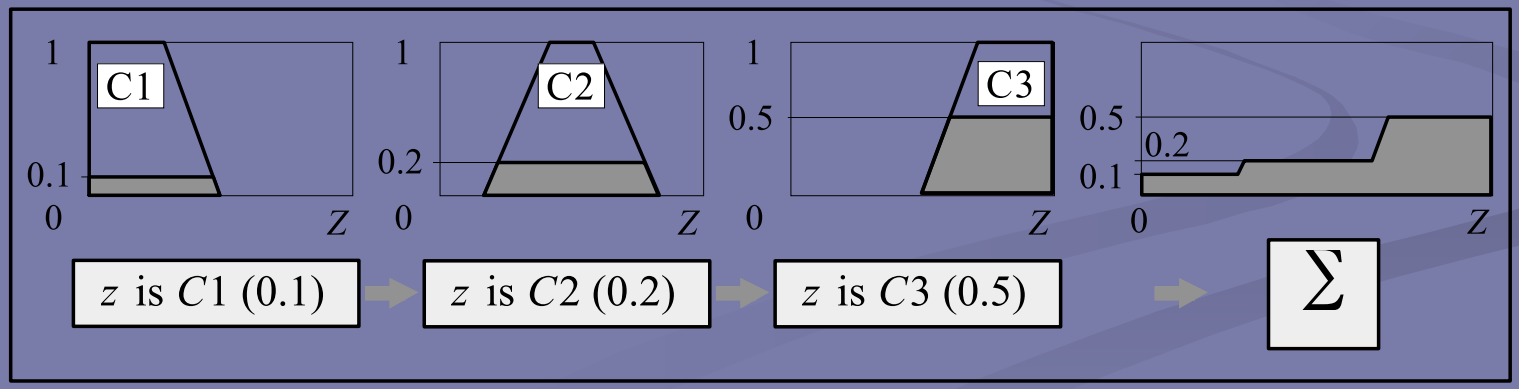


Step 2: Rule Evaluation

- The most common method of correlating the rule consequent with the truth value of the rule antecedent is to cut the consequent membership function at the level of the antecedent truth. This method is called **clipping** (alpha-cut).
- Since the top of the membership function is sliced, the clipped fuzzy set loses some information.
- However, clipping is still often preferred because it involves less complex and faster mathematics, and generates an aggregated output surface that is easier to defuzzify.
- While clipping is a frequently used method, **scaling** offers a better approach for preserving the original shape of the fuzzy set.
- The original membership function of the rule consequent is adjusted by multiplying all its membership degrees by the truth value of the rule antecedent.
- This method, which generally loses less information, can be very useful in fuzzy expert systems.

Step 3: Aggregation of the rule outputs

- Aggregation is the process of unification of the outputs of all rules.
- We take the membership functions of all rule consequents previously clipped or scaled and combine them into a single fuzzy set.
- The input of the aggregation process is the list of clipped or scaled consequent membership functions, and the output is one fuzzy set for each output variable.



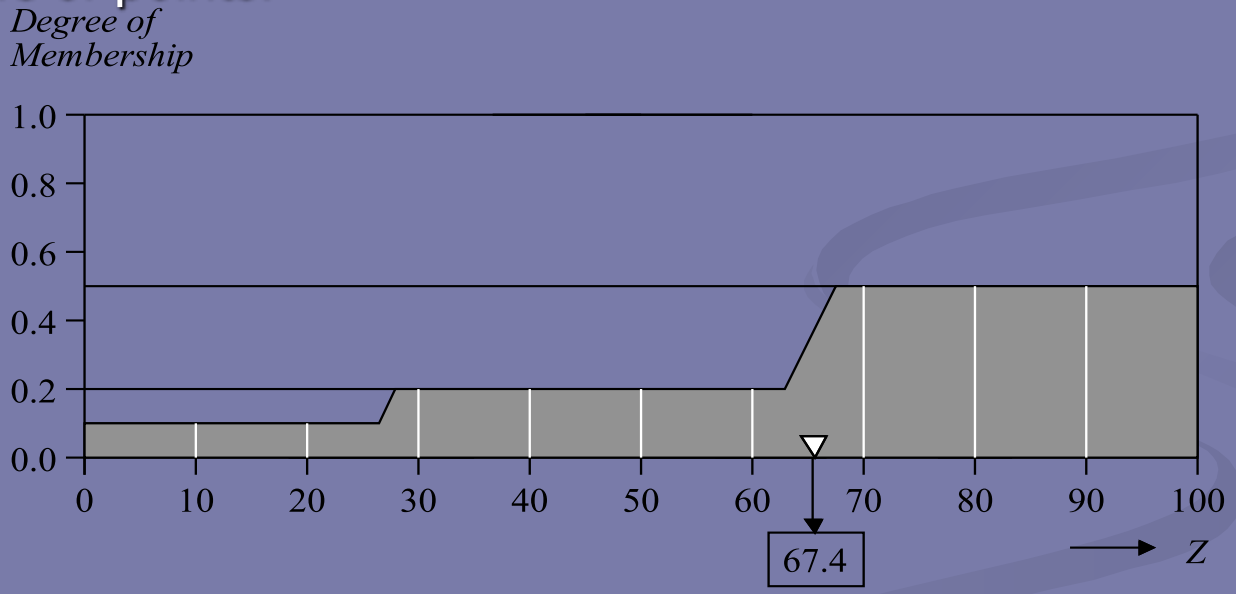
Step 4: Defuzzification

- The last step in the fuzzy inference process is defuzzification.
- Fuzziness helps us to evaluate the rules, but the final output of a fuzzy system has to be a crisp number.
- The input for the defuzzification process is the aggregate output fuzzy set and the output is a single number.
- There are several defuzzification methods, but probably the most popular one is the **centroid technique**. It finds the point where a vertical line would slice the aggregate set into two equal masses. Mathematically this **centre of gravity** (**COG**) can be expressed as:

$$COG = \frac{\int_a^b \mu_A(x)\, x\, dx}{\int_a^b \mu_A(x)\, dx}$$

Step 4: Defuzzification

- Centroid defuzzification method finds a point representing the centre of gravity of the aggregated fuzzy set A, on the interval [a, b].
- A reasonable estimate can be obtained by calculating it over a sample of points.



$$COG = \frac{(0+10+20)\times 0.1 + (30+40+50+60)\times 0.2 + (70+80+90+100)\times 0.5}{0.1+0.1+0.1+0.2+0.2+0.2+0.2+0.5+0.5+0.5+0.5} = 67.4$$

Sugeno Fuzzy Inference

- Mamdani-style inference, as we have just seen, requires us to find the centroid of a two-dimensional shape by integrating across a continuously varying function. In general, this process is not computationally efficient.

- Michio Sugeno suggested to use a single spike, a singleton, as the membership function of the rule consequent.

- A singleton, or more precisely a fuzzy singleton, is a fuzzy set with a membership function that is unity at a single particular point on the universe of discourse and zero everywhere else.

Sugeno Fuzzy Inference

- Sugeno-style fuzzy inference is very similar to the Mamdani method. Sugeno changed only a rule consequent. Instead of a fuzzy set, he used a mathematical function of the input variable. The format of the **Sugeno-style fuzzy rule** is

IF x is A AND y is B THEN z is f(x, y)

where:

x, y and z are linguistic variables;

A and B are fuzzy sets on universe of discourses X and Y, respectively;

f (x, y) is a mathematical function.

Sugeno Fuzzy Inference

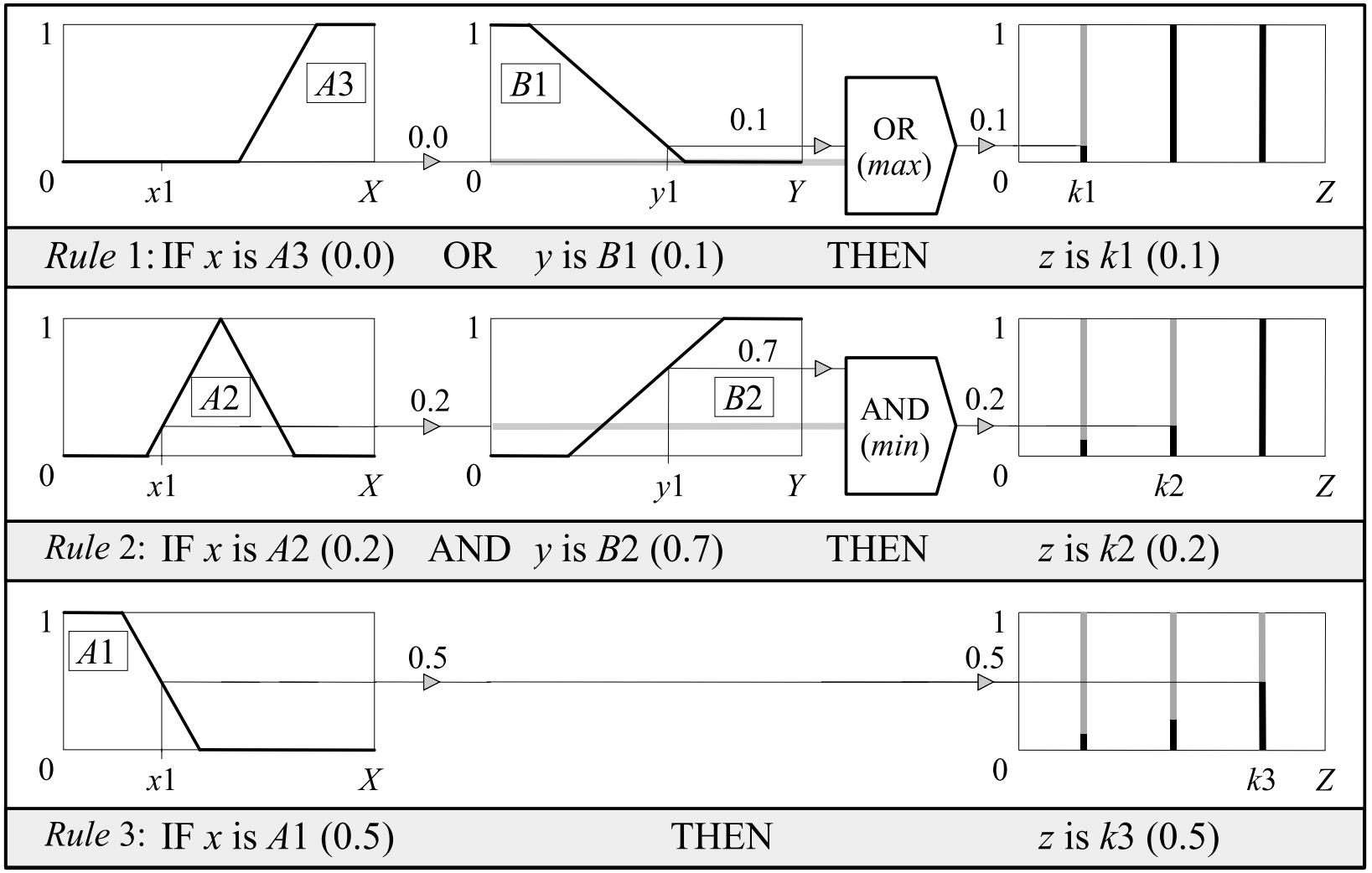
- The most commonly used **zero-order Sugeno fuzzy model** applies fuzzy rules in the following form:

  IF x is A AND y is B THEN z is k

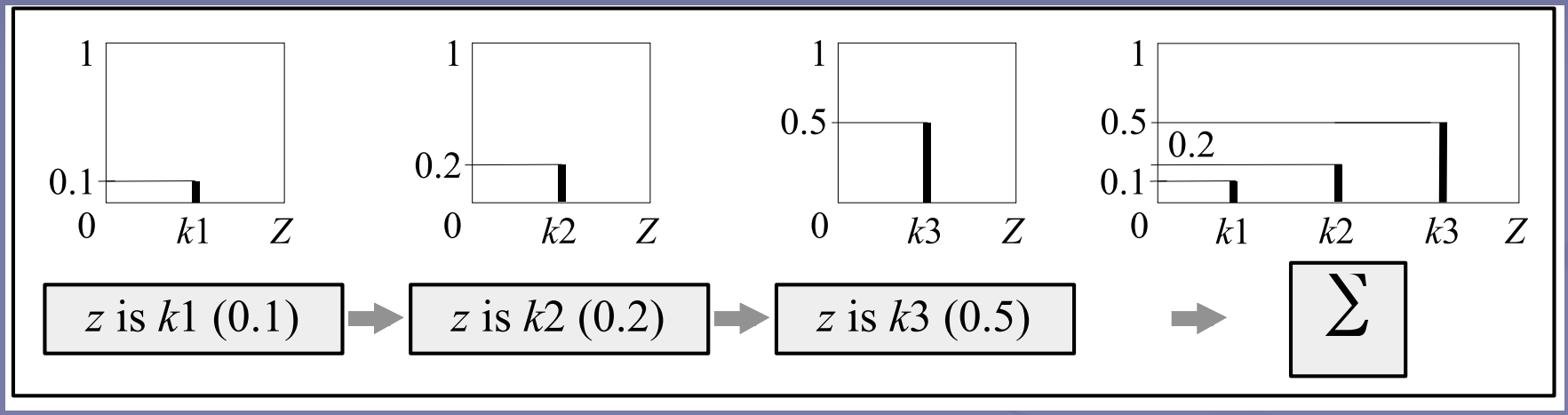
  where k is a constant.

- In this case, the output of each fuzzy rule is constant.
- All consequent **membership functions** are represented by **singleton spikes.**

Sugeno Rule Evaluation



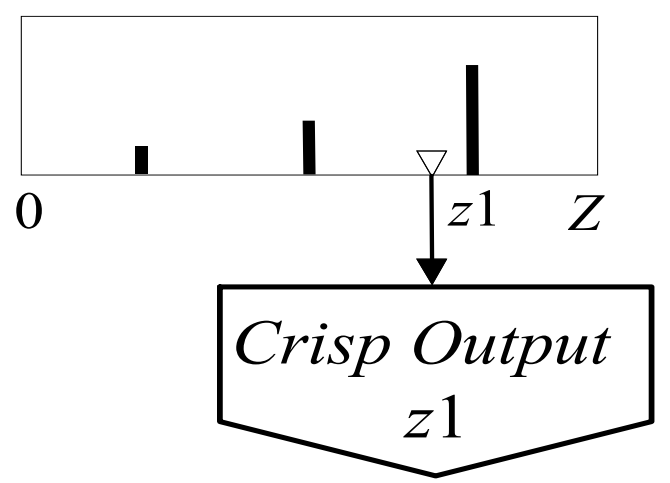
Sugeno Aggregation of the Rule Outputs



Sugeno Defuzzification

Weighted Average (WA)

$$WA = \frac{\mu(k1)\times k1 + \mu(k2)\times k2 + \mu(k3)\times k3}{\mu(k1) + \mu(k2) + \mu(k3)} = \frac{0.1\times 20 + 0.2\times 50 + 0.5\times 80}{0.1 + 0.2 + 0.5} = 65$$



Mamdani or Sugeno?

- Mamdani method is widely accepted for capturing expert knowledge. It allows us to describe the expertise in more intuitive, more human-like manner. However, Mamdani-type fuzzy inference entails a substantial computational burden.

- On the other hand, Sugeno method is computationally effective and works well with optimization and adaptive techniques, which makes it very attractive in control problems, particularly for dynamic nonlinear systems.